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Flow over Plates with Suction through Porous Strips

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Abstract

THIS paper addresses the steady, incompressible, two-dimensional flow past a flat plate with suction through porous strips. Closed-form solutions for each flow quantity are developed in the context of linearized triple-deck theory using Fourier transforms. To demonstrate the validity of these closed-form solutions, we compare the wall shear stress and pressure coefficients and the streamwise velocity profiles from the linearized theory with those obtained by the numerical integration of both interacting and nonsimilar boundary-layer equations. The agreement between the linearized triple-deck and interacting boundary-layer equations is good; however, the nonsimilar boundary layers, which fail to account for upstream influence, are shown to be in poor agreement with both interacting boundary layers and the linearized triple deck. The linearized closed-form solutions will therefore be very useful in future stability calculations.

Contents

Composite Solution for n Strips

We consider n porous strips centered at $x_1^*, x_2^*, \ldots, x_n^*$ ordered so that $x_1^* < x_2^* < \ldots < x_n^*$. We define the Reynolds number at strip i as

$$Re_{\infty i} = x_i^* U_\infty^* / \nu_\infty^*$$

Neglecting the influence of all downstream strips, we propose the dimensional flow quantities, denoted by *, in the neighborhood of the nth strip to be

$$\frac{u^*}{U_\infty^*} = f'(\eta) + \sum_{i=1}^{n} Re_{\infty i}^{1/4} \lambda^{-1/2} \frac{v^*_{\mathrm{wall}_i}}{U_\infty^*} \left[\left(\frac{f''(\eta)}{\lambda\sqrt{2}} - 1 \right) \delta \right.$$
$$\left. \times \left(\lambda^{5/4} \frac{x^* - x_i^*}{x_i^*} Re_{\infty i}^{3/8} \right) + \bar{u} \left(\lambda^{5/4} \frac{x^* - x_i^*}{x_i^*} Re_{\infty i}^{3/8}, y^* \frac{Re_{\infty i}^{5/8}}{x_i^*} \lambda^{3/4} \right) \right] \tag{1}$$

$$\frac{v^*}{U_\infty^*} = \sum_{i=1}^{n} Re_{\infty i}^{1/8} \lambda^{-1/4} \frac{v^*_{\mathrm{wall}_i}}{U_\infty^*} \left\{ [1 - f'(\eta)] \delta' \left(\lambda^{5/4} \frac{x^* - x_i^*}{x_i^*} Re_{\infty i}^{3/8} \right) \right.$$
$$\left. - \frac{y^*}{\pi} \int_{-\infty}^{\infty} \frac{\delta' \left(\lambda^{5/4} \frac{t - x_i^*}{x_i^*} Re_{\infty i}^{3/8} \right)}{(x^* - t)^2 + y^{*2}} \mathrm{d}t \right\} \tag{2}$$

$$\frac{p^* - p_\infty^*}{\rho_\infty^* U_\infty^{*2}} = \sum_{i=1}^{n} Re_{\infty i}^{1/8} \lambda^{-1/4} \frac{v^*_{\mathrm{wall}_i}}{U_\infty^*} \frac{1}{\pi} \int_{-\infty}^{\infty}$$
$$\times \frac{(x^* - t)\, \delta' \left(\lambda^{5/4} \frac{t - x_i^*}{x_i^*} Re_{\infty i}^{3/8} \right)}{(x^* - t)^2 + y^{*2}} \mathrm{d}t \tag{3}$$

where

$$\bar{u}(x,y) = \bar{u}_\infty (x - x_{\mathrm{LE}}, y) - \bar{u}_\infty (x - x_{\mathrm{TE}}, y) \tag{4}$$

$$\delta(x) = \delta_\infty (x - x_{\mathrm{LE}}) - \delta_\infty (x - x_{\mathrm{TE}}) \tag{5}$$

For $x < 0$,

$$\bar{u}_\infty = -\frac{9}{\pi\theta^2} \int_0^\infty \frac{\rho}{\rho^8 + 1} \int_0^{\rho\theta^{1/3} y} Ai(\eta)\, \mathrm{d}\eta\, e^{-\rho^3 \theta |x|} \mathrm{d}\rho \tag{6}$$

$$\delta_\infty = -\frac{3}{\pi\theta^2} \int_0^\infty \frac{\rho}{\rho^8 + 1} e^{-\rho^3 \theta |x|} \mathrm{d}\rho \tag{7}$$

For $x > 0$,

$$\bar{u}_\infty = -\frac{3}{2\pi i} \frac{|x|^{2/3}}{\theta^{4/3}} \int_0^{\infty e^{i\pi}} \frac{e^\tau}{\tau^{5/3}} \int_0^{(\tau^{1/3} y)/|x|^{1/3}} Ai(\eta)\, \mathrm{d}\eta\, \mathrm{d}\tau$$
$$+ \int_{\infty e^{-i\pi}}^0 \frac{e^\tau}{\tau^{5/3}} \int_0^{(\tau^{1/3} y)/|x|^{1/3}} Ai(\eta)\, \mathrm{d}\eta\, \mathrm{d}\tau$$
$$+ \frac{9}{4\pi\theta^2} \int_0^\infty \frac{\rho(1 - \sqrt{3}\rho^4)}{\rho^8 - \sqrt{3}\rho^4 + 1} \int_0^{\rho\theta^{1/3} y} Ai(-\eta)\, \mathrm{d}\eta\, e^{-\rho^3 \theta |x|} \mathrm{d}\rho$$
$$+ \frac{9}{4\pi\theta^2} \int_0^\infty \frac{\rho(\sqrt{3} - \rho^4)}{\rho^8 - \sqrt{3}\rho^4 + 1} \int_0^{\rho\theta^{1/3} y} Bi(-\eta)\, \mathrm{d}\eta\, e^{-\rho^3 \theta |x|} \mathrm{d}\rho \tag{8}$$

$$\delta_\infty = -\frac{|x|^{2/3}}{\theta^{4/3} \Gamma(5/3)} - \frac{3}{2\pi\theta^2} \int_0^\infty \frac{\sqrt{3}\rho^5 - \rho}{\rho^8 - \sqrt{3}\rho^4 + 1} e^{-\rho^3 \theta |x|} \mathrm{d}\rho \tag{9}$$

Comparison of the Linearized Triple-Deck Solutions with Interacting Boundary-Layer Solutions

In order to confirm the validity of the linearized triple-deck equations, we compare their results with those of the interacting boundary-layer equations. The first numerical results given represent the following one-strip configuration:

$x_p^* = 30.48$ cm (1 ft), strip width $= 1.52$ cm (0.05 ft)

$Re_\infty = U_\infty^* x_p^* / \nu_\infty^* = 1.0 \times 10^5$, $v^*_{\mathrm{wall}} = -2.3 \times 10^{-4} U_\infty^*$

Figures 1 and 2 show the normalized shear and pressure coefficients, respectively, on the wall for the linearized triple deck and the interacting and nonsimilar boundary layers. The agreement between the linearized triple deck and interacting boundary layers is excellent with a maximum error in the wall shear of approximately 7% in a neighborhood of the strip. Figures 1 and 2 also show the inaccuracy of the nonsimilar boundary-layer equations due to neglecting the interaction of the viscous and inviscid flows.

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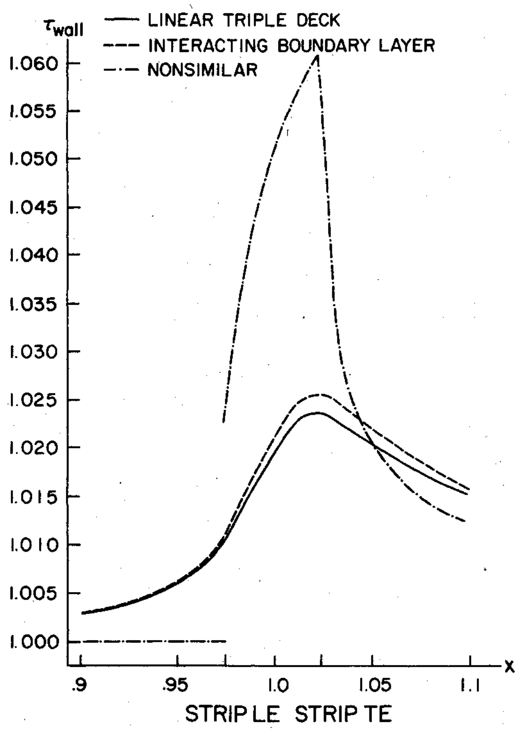

Fig. 1 Wall shear for one strip.

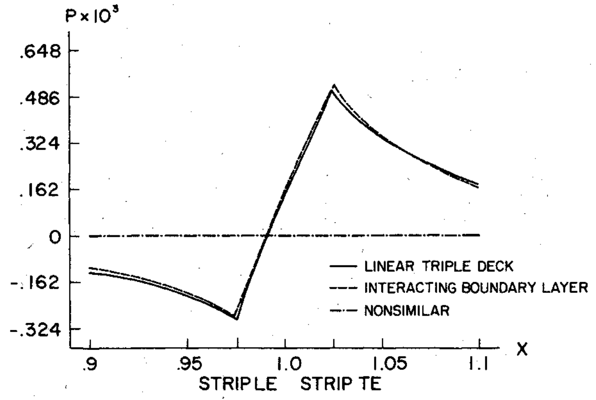

Fig. 2 Wall pressure coefficient for one strip.

In order to analyze the upstream and downstream influence predicted by the linearized theory, we compare the linearized triple deck with both the interacting and the nonsimilar boundary layers. For the same strip configuration as above, Fig. 3 shows that the upstream influence extends to about four strip widths. One strip width corresponds to about 16 reference boundary-layer thicknesses ($\delta_r = \sqrt{\nu^*_\infty x^* / U^*_\infty}$), so that four strip widths correspond to about 64 reference boundary-layer thicknesses. The upstream influence is practically the same for both the triple deck and the interacting boundary layers, in contrast with the nonsimilar boundary layers which predict zero upstream influence.

Figure 4 shows that the downstream influence extends more than 10 strip widths, corresponding to about 160 reference boundary-layer thicknesses. For the linearized triple deck, the decay of the strip's influence is algebraically slower than that predicted by the interacting boundary layers. This discrepancy is inherent in the linearized model. Our assumptions of streamwise variations in the flow quantities being $0(Re^{-3/8})$ and the strip's influence occurring in a neighborhood $0(Re^{-3/8} x^*_p)$ of the strip break down when we move far downstream of the strip.

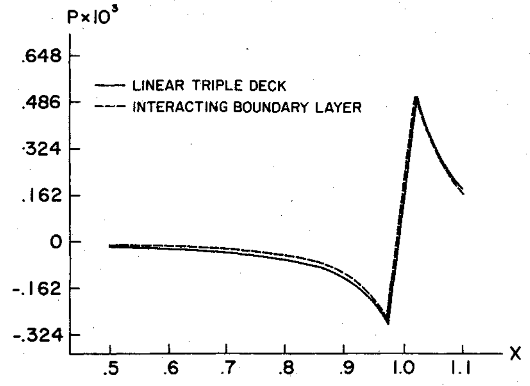

Fig. 3 Upstream influence (wall pressure coefficient).

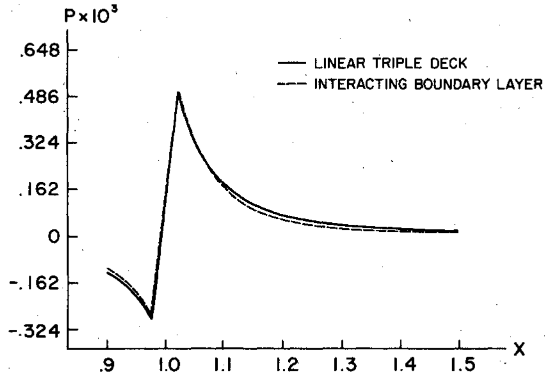

Fig. 4 Downstream influence (wall pressure coefficient).

Reference 1 contains a detailed mathematical account of the solution as well as a literature review. It also contains numerical results for the upstream and downstream influence on the shear for one and six strips. It compares the flowfield profiles. In all cases, the linearized triple-deck results are in very good agreement with those of the interacting boundary layers for the wall pressure coefficient, the wall shear, and the streamwise velocity component. Hence, one can confidently use the linearized triple-deck equations to solve accurately for the mean flow over a body with suction through porous strips.

Acknowledgments

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References

[1] Nayfeh, A. H., Reed, H. L., and Ragab, S. A., "Flow over Plates with Suction Through Porous Strips," AIAA Paper 80-1416, July 1980.